The curse of dimensionality of decision-making units: A simple approach to increase the discriminatory power of data envelopment analysis

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Abstract

Data envelopment analysis (DEA) is a technique for identifying the best practices of a given set of decision-making units (DMUs) whose performance is categorized by multiple performance metrics that are classified as inputs and outputs. Although DEA is regarded as non-parametric, the sample size can be an issue of great importance in determining the efficiency scores for the evaluated units, empirically, when the use of too many inputs and outputs may result in a significant number of DMUs being rated as efficient. In the DEA literature, empirical rules have been established to avoid too many DMUs being rated as efficient. These empirical thresholds relate the number of variables with the number of observations. When the number of DMUs is below the empirical threshold levels, the discriminatory power among the DMUs may weaken, which leads to the data set not being suitable to apply traditional DEA models. In the literature, the lack of discrimination is often referred to as the "curse of dimensionality". To overcome this drawback, we provide a simple approach to increase the discriminatory power between efficient and inefficient DMUs using the well-known pure DEA model, which considers either inputs only or outputs only. Three real cases, namely printed circuit boards, Greek banks, and quality of life in Fortune's best cities, have been discussed to illustrate the proposed approach..

Keywords: Data envelopment analysis, performance, printed circuit boards, banking, best

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cities.

1. Introduction

Data envelopment analysis (DEA) is an excellent management science tool that measures the relative performance of a set of entities or decision-making units (DMUs) with multiple performance measures that are classified as inputs and outputs. Nevertheless, problems of discrimination between efficient and inefficient DMUs often arise when there is a relatively large number of performance measures (variables) when compared to the number of DMUs; this may lead to efficient units being incorrectly classified as inefficient and inefficient units being misclassified as efficient. As Adler and Yazhemsky (2010) showed, "the latter occurs particularly frequently with small data sets under the assumption of variable returns-to-scale" (p. 283). In the literature, the lack of discrimination is often referred to as the "curse of dimensionality" (e.g., Daraio & Simar, 2007; Adler & Golany, 2007). The lack of discriminating power has important implications, as in practice it can limit the managerial insights that can be drawn (Ghasemi, Ignatius, & Rezaee, 2019).

In this sense, regarding the number of DMUs (sample size), it is quite clear that there are advantages to having larger data sets, as at a given number of DMUs, the efficiency score of each DMU can rely heavily on the number of variables (inputs and outputs) (Cinca & Molinero, 2004) – as such, the greater the number of variables, the less discerning the DEA analysis is (Jenkins & Anderson, 2003). Nevertheless, the literature indicates some empirical rules regarding the number of DMUs versus the number of inputs and outputs. For example, Golany and Roll (1989) and Homburg (2001) suggest that the number of DMUs should be at least twice the number of inputs and outputs. Nunamaker (1985), Banker et al. (1989), Friedman and Sinuany-Stern (1998), and Raab and Lichty (2002) suggest that the number of DMUs should be at least three times the number of inputs and outputs; and Dyson et al. (2001) suggest that the number of DMUs should be at least twice the product of the number of inputs and the number of outputs. Yet, another empirical rule of thumb which can provide guidance is, in line with Cooper, Seiford, and Tone (2007), $n \geq \max(m \text{ x } s, 3(m+s))$, where n is the number of DMUs, m is the number of inputs, and s is the number of outputs.

Figure 1 shows the number of DMUs that would be required in the case of each empirical rule of thumb mentioned above. The observation to be made is that, even for the case of 12 inputs and 12 outputs, the number of DMUs required becomes very high, ranging between 48 and 288, depending on the rule of thumb used. This may turn out to be a problem in real-life applications, where a high number of DMUs may simply just not be available.

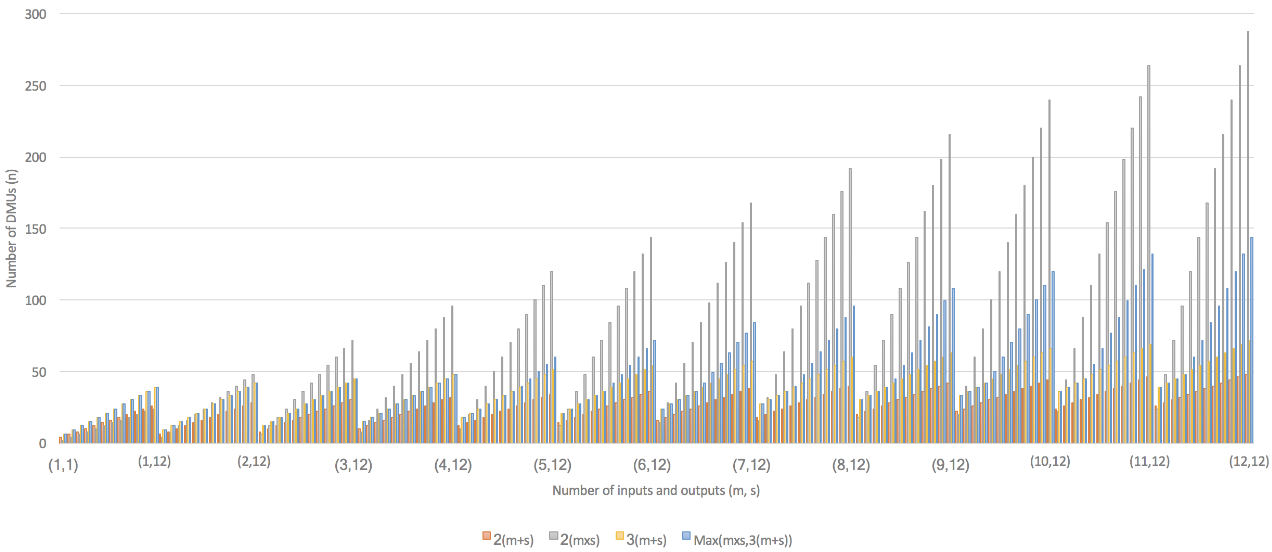

Figure 1: Comparison of the empirical rules of thumb.

It should be noted that Cook, Tone, and Zhu (2014) stated that, whereas in statistical regression analysis the sample size is a vital issue, as it tries to estimate the average behavior of a set of DMUs, when used as a benchmarking tool, DEA focuses on the performance of each DMU, and as such, the sample size or the number of DMUs being evaluated may be immaterial. The issue remains, however, that when the number of DMUs goes below the threshold levels, the discriminatory power among the DMUs may weaken.

Despite the above-mentioned thumb rules, there are many studies in performance measurement that do not meet them, but which, nevertheless, apply DEA methods (for examples, the interested reader is referred to the studies by Adler & Golany, 2001; Liang, Li, & Li, 2009; Ragsdale, 2006; Sarkis, 2000; Wagner & Shimshak, 2007; and Wong & Beasley, 1990, among others). The rationale behind "breaking" the empirical rules is that there are practical conditions that may lead the analyst to choose as many variables as possible (Xie, Dai, Li, & Jiang, 2014) - among these, the fact that any resource used in the process should be treated as an input (Wagner & Shimshak, 2007) or that even if the analyst would con-sider reducing the number or omitting some of the variables (Pastor, Ruiz, & Sirvent, 2002), given the complexity and the interrelationships among the variables, this may turn out to be very difficult to carry out. Naturally, the question that arises under these circumstances is: How can the discriminatory power of DEA be increased? In light of the above discussion, the potential practical implications of asking such a question are rather obvious.

The research studies in the existent literature that were devoted to improving the discriminatory power of DEA between efficient and inefficient DMUs can be classified into two categories: (a) the ones that are aimed at increasing the number of DMUs, while maintaining the same number of variables; and (b) the ones that are aimed at reducing the number of variables used. The first category generally uses pooled cross section and time series data; nevertheless, this approach has a major drawback in the sense that it assumes no technological change over the sample periods (Hughes & Yaisawarng, 2004). The second category generally uses variable reduction based on a partial covariance analysis (see for example, Jenkins & Anderson, 2003) or principal component analysis combined with DEA (see, for example, Adler & Golany, 2001, 2002, 2010); nevertheless, this approach suffers from a major drawback surfaced from the loss of variable information and negative numbers.

Some studies have further incorporated value judgments of decision-makers into DEA models via weights restrictions (Allen et al., 1997) or preference change methods (Meng et al., 2008; Zhang et al., 2009); nevertheless, these approaches rely on expert opinion, which is not always easy or feasible to obtain (Doyle & Green, 1994). Others have used cross-efficiency (Doyle & Green, 1994), but this method suffers from non-uniqueness of the DEA optimal input-output weights; also, it can generate negative efficiencies (Wu, Liang, & Chen, 2009). There are also research studies that were devoted to discriminating among the efficient DMUs and these employed super-efficiency models (Andersen & Petersen, 1993), wherein the DMU under evaluation is excluded from the sample and is then evaluated with respect to the new production possibility set created by other DMUs. Super-efficiency, however, may result in infeasibility of envelopment models or unboundedness of multiplier models (Seiford & Zhu, 1999). Other approaches to increasing the discrimination of DEA include the use of the distances to both the efficient frontier and the anti-efficient frontier (Shen et al., 2016), reference frontier share (Rezaeiani & Foroughi, 2018), and deviation variables framework (Ghasemi, Ignatius, & Rezaee, 2019), among others. In this paper, we explore another idea to enhance the discriminatory power of DEA, more specifically between efficient and inefficient DMUs, using variable reduction based on the pure DEA model. It should be mentioned that, by definition, DEA models have both inputs and outputs; and pure DEA refers to a class of models wherein either inputs only or outputs only are con-sidered (see, for example, Lovell & Pastor, 1999; and Seiford & Zhu, 1998). There is yet another approach that also considers only inputs or only outputs, which is widely known as the Benefit-of-the-Doubt (BoD), proposed by Cherchye et al. (2004, 2007) for building composite indicators from a set of subindicators. The BoD model tries to summarize all subindicators in an overall aggregated indicator (the DEA score). The pure DEA model and the BoD are equivalent models.

In this paper, we consider the empirical thresholds proposed in the literature that relate the number of variables with the number of observations. As previously mentioned, when the number of DMUs is below the empirical threshold levels, the discriminatory power among the DMUs may weaken, which can lead to categorizing a large number of DMUs as best practice or efficient. We are thus interested in investigating the situation wherein a large number of DMUs are deemed as efficient, when this may not necessarily be the case (Adler & Yazhemsky, 2010). Furthermore, such a situation is also not desirable by the user, and is caused by the relatively large number of performance measures (I/O variables) when compared to the number of DMUs. To overcome this drawback, we propose a simple approach wherein we use an output-oriented envelopment model without explicit inputs to collapse the output variables in an input-oriented envelopment model and the input variables in an output-oriented envelopment model.

The remainder of the paper is organized as follows. In the next section, we build upon the introduction section and showcase the motivation of the study with an example. We then propose a simple approach using the well-known pure DEA model to increase the discriminatory power between efficient and inefficient DMUs; we investigate the cases of the output variable reduction for the input-oriented envelopment model and the input variable reduction for the output-oriented envelopment model. Subsequently, we discuss three real cases to illustrate the proposed approach. The final section concludes the paper.

2. Motivating Example

A simple example will provide context for our proposed approach. It will also help justify the key elements of our approach and motivate the main steps in the analysis.

Consider a set of n DMUs, which has m inputs and s outputs, where the input and output vector of each DMU_j ($j = 1, ..., n$) is $\text{X}_j = (x_{1j}, ..., x_{mj})$ and $\text{Y}_j = (y_{1j}, ..., y_{sj})$, respectively. The performance of the DMUs is measured based on the given inputs and outputs, through the following linear programming model, which is widely known as the envelopment form of the input-oriented DEA model under variable returns to scale (VRS) (Banker et al., 1984):

$$
\begin{aligned}
&\min \theta \\
&\text{subject to} \\
&\sum_{j=1}^{n} x_{ij}\lambda_j \leqq \theta x_{io}, \quad i = 1, 2, \ldots, m, \\
&\sum_{j=1}^{n} y_{rj}\lambda_j \geqq y_{ro}, \quad r = 1, 2, \ldots, s, \\
&\sum_{j=1}^{n} \lambda_j = 1, \\
&\lambda_j \geqq 0, \quad j = 1, 2, \ldots, n,
\end{aligned}
\tag{1}
$$

where λ_j is the j^{th} intensity variable and θ is the measure of performance, which ranges from zero to one.

For a better understanding of the previous discussion on the discriminatory power problem, let us work out the following simple example. Assume that we are interested in measuring the performance of five DMUs, namely A, B, C, D, and E, which have one single input, x_1, and three outputs, y_1, y_2, and y_3 (see Table 2).

Table 1: Original Sub-Indexes

DMU	x_1	y_1	y_2	y_3	θ	Super-Eff.
A	10	10	20	30	1	Infeasible
B	10	10	30	20	1	Infeasible
C	30	30	10	20	1	Infeasible
D	30	30	20	10	1	Infeasible
E	10	10	10	10	1	1

According to Model (1), all the five DMUs are efficient, which can be seen in the penultimate column of Table 2. It is also to be clearly noted that the sample size of 5 DMUs is smaller than the empirical threshold levels discussed in the literature (see Section 1).

Figure 2 provides a snapshot of the frontiers based on the given input x_1 versus every single output $y_r, \forall r$. The three figures (a) - (c) in Figure 2 represent the frontiers for (x_1, y_1), (x_1, y_2), and (x_1, y_3), respectively; from the figures, one can infer that the DMUs that are not on the frontier are deemed to be inefficient. In other words, we can say that it is the total number of outputs (i.e., 3) that causes all of the DMUs to be efficient in Table 2.

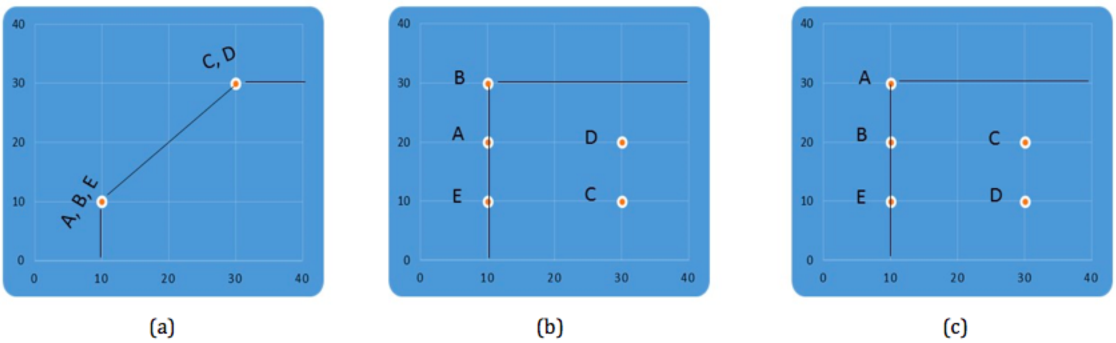

Figure 2: Input versus every single output.

In particular, it is worth mentioning that units C and D could be seen as "partially" inefficient since they appear in the interior of the technology in figure 2(b) and figure 2(c). Regarding unit E, this is Pareto-dominated by units A and B in figure 2(b) and figure 2(c). As we will show later in the paper, once a new discriminant score is introduced, units C, D, and E in this example will be identified as inefficient.

Super-efficiency models were introduced in DEA in order to discriminate the performance among efficient DMUs (Andersen & Petersen, 1993). In traditional super-efficiency DEA models, the DMU under evaluation is excluded from the reference set in the constraints of Model (1). Notice that, in our example, if we apply the super-efficiency model for discriminating among the efficient units (all the DMUs in this instance), we get infeasibility for units A, B, C and D, whereas unit E keeps being efficient (see last column in Table 2) —something that is usual under VRS. So, super-efficiency seems not be the solution for this type of situations where the sample size and the number of variables is unbalanced.

3. Modelling

In this section, we carry on to show the steps involved in our proposed approach, assuming that the model under evaluation is weakly over-determined. First, we introduce Model (2), which represents the base model that we will further use to collapse the inputs and/or outputs. We will also provide a discussion of the input and output grouping rationale.

Since the aim of this paper is more of a pragmatic nature, we approach the matter as a design problem (see, for example, Wieringa, 2014), wherein we propose a simple artefact (model) that better contributes to the achievement of a goal. Design science research produces artefacts (Geerts, 2011), wherein artefacts, in the regular common understanding of the term, are defined as human-made objects, usually for practical purposes.

Design science artefacts should possess two essential characteristics: relevance and novelty (Geerts, 2011; Hevner, March, Park, & Ram, 2004); the artefact that we proceed to introduce in this paper has both. On the one hand, it is relevant, as it solves the ongoing problem of the lack of discriminatory power that can arise when using DEA models on particular data sets. On the other hand, it is novel in the sense that although previous attempts have been made to increase the discriminatory power of DEA, these have various shortcomings (as discussed in Section 1) and so it can be said that the present artefact addresses a solved problem in a more effective way. In line with the above discussion, the design problem can be formulated as follows: *Increase the discriminatory power among the DMUs by designing an approach that yields a ranking with more discriminant ability for individual DMUs in order to measure performance.*

Peffers, Tuunanen, Rothenberger, and Chatterjee (2008) introduced the Design Science Research Methodology (DSRM), consisting of a nominal sequence of activities to be fol-lowed in the process of creating an artefact; in Table 1, we discuss the activities that are relevant in the context of the present study. The first column lists the DSRM activities and the second column describes each of these activities; finally, the third column indicates the materials from and through which the activities are executed, such as models, methods, and foundational theories, instruments and frameworks, among others (Hevner, March, Park, & Ram, 2004).

Table 2: Design Science Research Methodology (DSRM) applied to the Current Study

DSRM activities	Activity description	Knowledge base
Problem identification and motivation	Failure to evaluate performance correctly due to lack of discrimination among efficient and inefficient DMUs because of the relatively large number of variables when compared to the number of DMUs.	Literature review. Understanding of weaknesses of traditional DEA models. Real world problem.
Define the objectives of a solution	Design of an approach that yields a ranking with more discriminant ability for individual DMUs.	Literature review. Knowledge of existing tools.
Design and development	Design of an approach (use of an existing DEA model) that is able to collapse the inputs and outputs into a single input (or fewer inputs) and single output (or fewer outputs), respectively; or which can allow the collapse to take place simultaneously in both the inputs and outputs side.	Pure DEA model.
Demonstration	Case study demonstration. The proposed approach is used to collapse the inputs and/or outputs for 3 different data sets.	Applying the proposed approach to three real-world cases.
Evaluation	Comparative analysis.	Understanding of current solution and its advantages.

3.1. An envelopment model without explicit inputs

An output-oriented envelopment problem without explicit inputs can be formulated, in line with Lovell and Pastor (1999), as follows:

$$
\begin{aligned}
&\delta^* = \min \sum_{j=1}^{n} \delta_j \\
&\text{subject to} \\
&\sum_{j=1}^{n} \Upsilon_{lj}\delta_j \geqq \Upsilon_{lo}, \quad l = 1, 2, \ldots, k, \\
&\delta_j \geqq 0,\, j = 1, 2, \ldots, n,
\end{aligned}
\tag{2}
$$

where Υ_{lo} is the l^{th} output of the unit being evaluated, Υ_{lj} is the l^{th} output of the j^{th} unit, δ_j is the j^{th} intensity variable, δ^* is the optimal value of Model (2), and k is the number of variables under study. Note that Model (2) has one less variable (the radial efficiency score) and one less constraint (the convexity constraint) when compared to the following standard output-oriented envelopment problem without explicit inputs:

$$
\begin{aligned}
&\max\ \alpha \\
&\text{subject to} \\
&\sum_{j=1}^{n} \Upsilon_{lj}\gamma_j \geqq \alpha\Upsilon_{lo}, \quad l = 1, 2, \ldots, k, \\
&\sum_{j=1}^{n} \gamma_j = 1, \\
&\gamma_j \geqq 0,\, j = 1, 2, \ldots, n.
\end{aligned}
\tag{3}
$$

where γ_j is the j^{th} intensity variable.

The formulations of Model (2) and Model (3) show that, from a mathematical point of view, they do not consider inputs at all; hence, they are referred to as pure output-oriented envelopment models. One could derive the value of the efficiency score as $\alpha^* = \delta^{*-1} = (\sum_{j=1}^{n} \delta_j)^{-1}$.

The geometrical interpretation of Model (2), in line with Lovell and Pastor (1999), could be done as follows: for all $0 < \delta_j, \forall j \in \{1, 2, ..., n\}$, α^* being the efficiency score, in the absence of slacks, we can state that $\Upsilon_{lo} = \sum_{j=1}^{n} \Upsilon_{lj}\delta_j, \forall l \in \{1, 2, ..., k\}$, and consequently that $\Upsilon_{lo}\alpha^* = \Upsilon_{lo}\delta^{*-1} = \sum_{j=1}^{n} \Upsilon_{lj}\delta_j\delta^{*-1}, \forall l \in \{1, 2, ..., k\}$, which clearly indicates that the projection of Υ_o onto the efficient frontier, $\Upsilon_o\alpha^*$, is a convex combination of Υ_j's. The optimization of Model (2) leads to the selection of the rays defined by the efficient units which are jointly as close as possible to the ray defined by Υ_o. In other words, optimising Model (2) allows us to search for the smallest cone defined by the rays associated with efficient units that is capable to generate the ray associated with the DMU of interest; the generating rays of the cone correspond to the peers for the DMU of interest.

3.2. Output variable reduction in an input-oriented envelopment model

Consider that Model (1) suffers from the curse of dimensionality, which results in a weakened discriminatory power between efficient and inefficient DMUs. To avoid such a problem, let us use the knowledge obtained from Model (2) (by considering $\Upsilon \equiv Y$ and $k = s$ in Model (2)) to obtain the following Model (4). Figure 3 graphically summarizes the said concept.

$$
\begin{aligned}
&\min \phi \\
&\text{subject to} \\
&\sum_{j=1}^{n} x_{ij}\lambda'_j \leqq \phi x_{io}, \quad i = 1, 2, \dots, m, \\
&\sum_{j=1}^{n} \delta^*_j \lambda'_j \geqq \delta^*_o, \\
&\sum_{j=1}^{n} \lambda'_j = 1, \\
&\lambda'_j \geqq 0, \quad j = 1, 2, \dots, n.
\end{aligned}
\tag{4}
$$

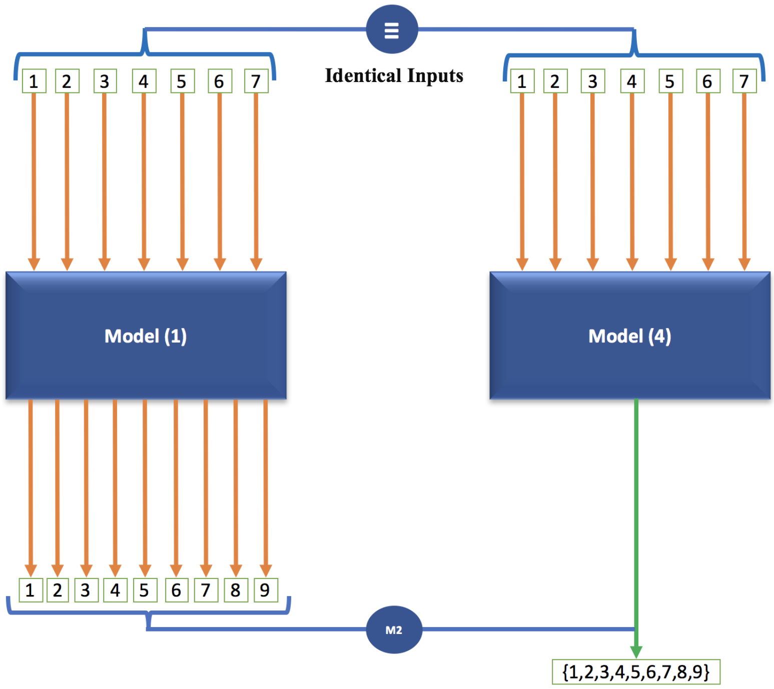

Figure 3: Collapsing all outputs into a single output.

It is to be noted that the s outputs have been collapsed into one single output score, δ^*, which is obtained from Model (2). The last but one constraint in Model (4) is formed based on this single output score. Model (4) is a modified version of Model (1), wherein s output constraints have been replaced with one single output constraint. Now, the input-oriented envelopment Model (4) counts with $m + 2$ constraints, along with the set of all n non-negativity conditions on $\lambda'_j, \forall j$. The discriminant score (D_S) of the j^{th} DMU is defined as follows:

$$D_S = \begin{cases} \phi_j \delta_j^*, & \text{if } max\{\phi_j \delta_j^*, \forall j\} = 1; \\ \frac{\phi_j \delta_j^*}{max\{\phi_j \delta_j^*, \forall j\}}, & \text{otherwise.} \end{cases} \tag{5}$$

Consider the input and outputs data provided in Table 3. Since all θs are equal to 1, this means that all DMUs are considered to be efficient. Let us first collapse the outputs y_1, y_2, and y_3 into δ^* using Model (2). Then, we can use the obtained δ^* in the output constraint of Model (4). Subsequently, by solving Model (4), we can obtain the penultimate column in Table 3. The last column of Table 3 shows the discriminant score and we can quickly observe that DMUs C, D, and E are inefficient; in conclusion, we have been able to increase the discriminatory power among the individual DMUs.

Table 3: Discriminatory Power among DMUs

DMU	x_1	y_1	y_2	y_3	θ	δ^*	ϕ	D_S
A	10	10	20	30	1	1	1	1
B	10	10	30	20	1	1	1	1
C	30	30	10	20	1	1	0.33	0.33
D	30	30	20	10	1	1	0.33	0.33
E	10	10	10	10	1	0.5	1	0.5

Output grouping rationale: The previous discussion in this section considered that all the outputs could be grouped together into a single output; however, in reality such an assumption may be crude: some of the outputs might group together while some others might not. It is to be noted, nonetheless, that the process of grouping the outputs is based on some rationale, such as an expert opinion or based on the literature reviewed. Let S be the set that consists of the sets of grouped and un-grouped outputs. Let $\Upsilon_g \equiv Y_g, \forall g \in G \setminus \{\}$, where G is the set that consists of the sets of grouped outputs only. For example, let there be nine outputs and let us say that as per the experts' opinion, the nine outputs could be grouped into $S = \{\{1\}, \{2, 4\}, \{3\}, \{5, 6, 7\}, \{8, 9\}\}$, then $G = \{\{2, 4\}, \{5, 6, 7\}, \{8, 9\}\}$ and $S \setminus G = \{\{1\}, \{3\}\}$. In order to collapse every subset of G, one has to use Model (2) repeatedly as many times as the number of subsets in $G \setminus \{\}$, considering $k = |g|$ for every g, where $|\ |$ represents the cardinality of the set. Figure 4 graphically summarizes the discussed concept.

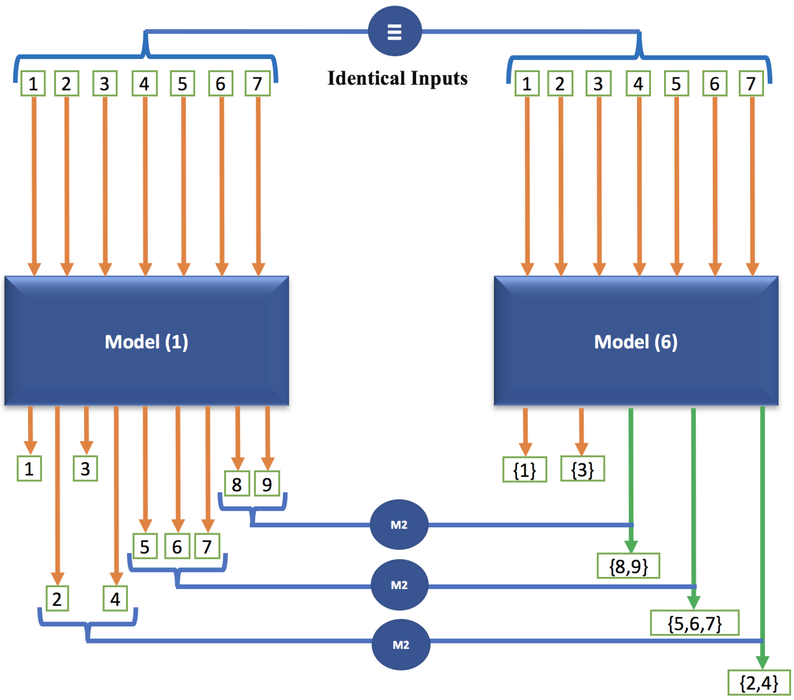

Figure 4: Collapsing outputs into fewer outputs.

The following Model (6) is the variant of Model (4). Model (4) has one output constraint, whereas Model (6) has $|S|$ output constraints, out of which $|G \setminus \{\}|$ are grouped constraints obtained based on Model (2) and $|S \setminus G|$ are un-grouped constraints.

$$
\begin{aligned}
&\min\ \phi \\
&\text{subject to} \\
&\sum_{j=1}^{n} x_{ij}\lambda'_j \leqq \phi x_{io}, \quad i = 1, 2, \ldots, m, \\
&\sum_{j=1}^{n} y'_{rj}\lambda'_j \geqq y'_{ro}, \quad r = 1, 2, \ldots, |S \setminus G|, \\
&\sum_{j=1}^{n} \delta^*_{g'j}\lambda'_j \geqq \delta^*_{g'o}, \quad g' = 1, 2, ..., |G \setminus \{\}|, \\
&\sum_{j=1}^{n} \lambda'_j = 1, \\
&\lambda'_j \geqq 0, \quad j = 1, 2, \ldots, n,
\end{aligned}
\tag{6}
$$

where y'_{rj} is the r^{th} un-grouped output of the j^{th} DMU and λ'_j is the j^{th} intensity variable.

Now, the input-oriented envelopment Model (6) counts with $m + |S| + 1$ constraints along with n non-negativity conditions. Note that since Model (6) is a variant of Model (4), then Model (6) may not yield identical scores. The scores of Model (6) depend on the grouping of the outputs, so the scores are dependent on the experts' strategy of grouping the outputs. The discriminant score of the j^{th} DMU is obtained by Equation (5), wherein $\delta^*_j = max(\delta^{\ *}_{g'j}, g' = 1, 2, ..., |G \setminus \{\}|), \forall j$.

3.3. Input variable reduction in an output-oriented envelopment model

First, let us consider the output-oriented envelopment model for VRS, as follows:

$$
\begin{aligned}
&\max\ \psi \\
&\text{subject to} \\
&\sum_{j=1}^{n} x_{ij}\lambda_j \leqq x_{io}, \quad i = 1, 2, \ldots, m, \\
&\sum_{j=1}^{n} y_{rj}\lambda_j \geqq \psi y_{ro}, \quad r = 1, 2, \ldots, s, \\
&\sum_{j=1}^{n} \lambda_j = 1, \\
&\lambda_j \geqq 0, j = 1, 2, \ldots, n.
\end{aligned} \tag{7}
$$

Consider that Model (7) suffers from the curse of dimensionality, which results in a weakened discriminatory power between efficient and inefficient DMUs. We can once again use the knowledge obtained from Model (2) (by considering $\Upsilon \equiv X$ and $k = m$ in Model (2)) to obtain the following Model (8). Figure 5 graphically summarizes the said concept.

$$
\begin{aligned}
&\max\ \varphi \\
&\text{subject to} \\
&\sum_{j=1}^{n} \delta_j^* \lambda_j' \leqq \delta_o^*, \\
&\sum_{j=1}^{n} y_{rj}\lambda_j' \geqq \varphi y_{ro}, \quad r = 1, 2, \ldots, s, \\
&\sum_{j=1}^{n} \lambda_j' = 1, \\
&\lambda_j' \geqq 0, \quad j = 1, 2, \ldots, n.
\end{aligned} \tag{8}
$$

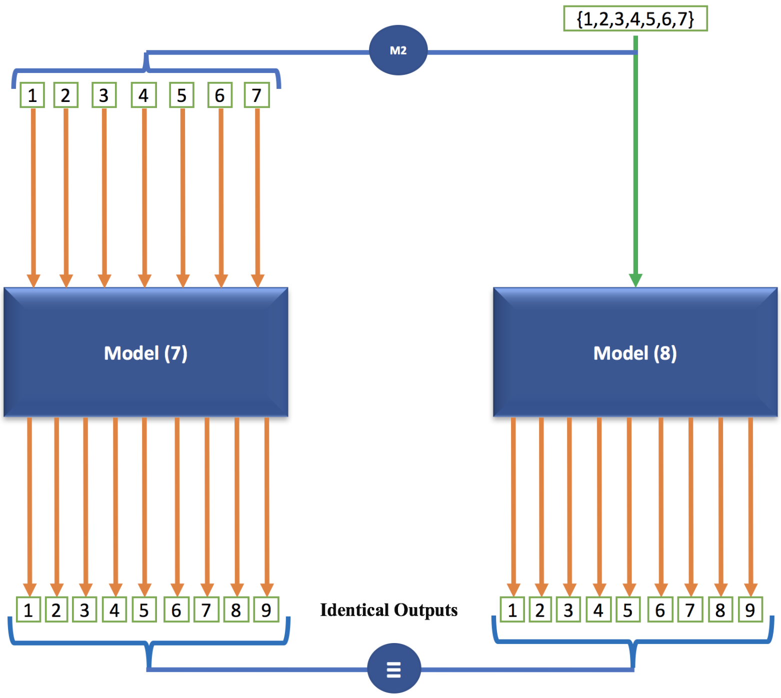

Figure 5: Collapsing all inputs into a single input.

Using Model (2), the m inputs have been collapsed into one single input score, δ^*. This single input score is further reflected in the first constraint of Model (8). As such, Model (8) is a modified version of Model (7), wherein m input constraints have been replaced with one single input constraint. Now, the output-oriented envelopment Model (8) counts with $s+2$ constraints, along with the set of all n non-negativity conditions on λ'_j, $\forall j$. The discriminant score of the j^{th} DMU is defined as follows:

$$D_S = \begin{cases} \varphi_j^{-1}\delta_j^*, & \text{if } max\{\varphi_j^{-1}\delta_j^*, \forall j\} = 1; \\ \frac{\varphi_j^{-1}\delta_j^*}{max\{\varphi_j^{-1}\delta_j^*, \forall j\}}, & \text{otherwise.} \end{cases} \tag{9}$$

To exemplify, let us consider the inputs and output data for five DMUs in Table 4. An analysis of the performance of the five DMUs, using Model (7), indicates that all the DMUs are efficient, since all the ψs are equal to 1 (see the 6th column in Table 4). We proceed to collapse the inputs x_1, x_2, and x_3 into δ^* using Model (2). Then, we feed the obtained δ^* in the input constraint of Model (8). We solve Model (8), the results of which are displayed in the penultimate column of Table 4. Finally, the discriminant score is calculated (see the last column of Table 4), which reveals that DMUs a, b, and e are actually inefficient. In other words, the proposed approach has been successfully applied to increase the discriminatory power among the individual DMUs.

Table 4: Discriminatory Power among DMUs

DMU	x_1	x_2	x_3	y_1		δ^*	φ^{-1}	D_S
a	30	10	20	10	1	1	0.33	0.33
b	20	10	30	10	1	1	0.33	0.33
c	20	30	10	30	1	1	1	1
d	10	30	20	30	1	1	1	1
e	10	10	10	10	1	0.5	1	0.5

Input grouping rationale: Similarly to the output grouping rationale discussed in the previous section, one could also collapse all or some of the inputs into fewer groups. In this case, also, the process of grouping the inputs is based on some rationale, such as an expert opinion or based on a review of existing literature. Let M be the set that consists of the sets of grouped and un-grouped inputs. Let $\Upsilon_f \equiv X_f, \forall f \in F \setminus \{\}$, where F is the set that consists of the sets of grouped inputs only. For example, let there be seven inputs and let us say that as per the experts' opinion, the seven inputs could be grouped into $M = \{\{1, 2, 3\},\{4, 6\}, \{5\}, \{7\}\}$, then $F = \{\{1, 2, 3\}, \{4, 6\}\}$ and $M \setminus F = \{\{5\}, \{7\}\}$. In order to collapse every subset of F, one has to use Model (2) repeatedly as many times as the number of subsets in $F \setminus \{\}$, considering $k = |f|$ for every f. Figure 6 graphically summarizes the discussed concept.

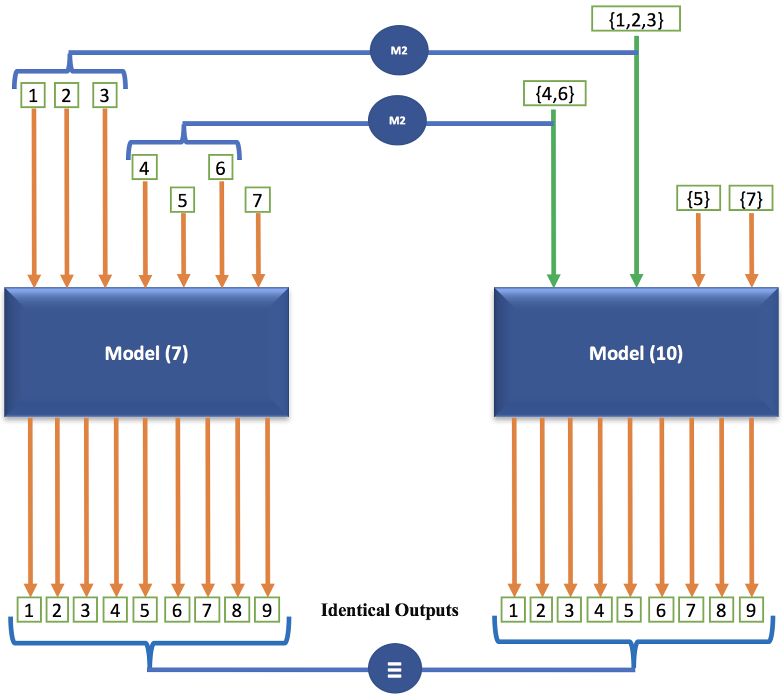

Figure 6: Collapsing inputs into fewer inputs.

The following Model (10) is the variant of Model (8). Model (8) has one input constraint, whereas Model (10) has $|M|$ input constraints, out of which $|F \setminus \{\}|$ are grouped constraints obtained based on Model (2) and $|M \setminus F|$ are un-grouped constraints.

$$
\begin{aligned}
&\max \varphi \\
&\text{subject to} \\
&\sum_{j=1}^{n} x'_{ij}\lambda'_j \leqq x'_{io}, \quad i = 1, 2, \ldots, |M \setminus F|, \\
&\sum_{j=1}^{n} \delta^*_{f'j}\lambda'_j \leqq \delta^*_{f'o}, \quad f' = 1, 2, ..., |F \setminus \{\}|, \\
&\sum_{j=1}^{n} y_{rj}\lambda'_j \geqq \varphi y_{ro}, \quad r = 1, 2, \ldots, s, \\
&\sum_{j=1}^{n} \lambda'_j = 1, \\
&\lambda'_j \geqq 0, j = 1, 2, \ldots, n.
\end{aligned}
\tag{10}
$$

where x'_{ij} is the i^{th} un-grouped input of the j^{th} DMU and λ'_j is the j^{th} intensity variable.

Now, the output-oriented envelopment Model (10) counts with $|M| + s + 1$ constraints along with n non-negativity conditions. Note that since Model (10) is a variant of Model (8), then Model (10) may not yield identical scores. The scores of Model (10) depend on the grouping of the inputs, so the scores are dependent on the experts' strategy of grouping the inputs. The discriminant score of the j^{th} DMU is obtained by Equation (9), wherein $\delta^*_j = max(\delta^*_{f'j}, f' = 1, 2, ..., |F \setminus \{\}|), \forall j$.

3.4. Input and output variable reduction in an input- or output-oriented envelopment model

The previous sections showed how the collapse can take place either on the input side or on the output side only. The following Model (11) and Model (12) are the input-oriented model and the output-oriented model, respectively, that allow for the collapse to take place in both the input and the output sides. The reader is referred to the the models introduced previously for notational explanations.

$$
\begin{aligned}
&\min \ \phi \\
&\text{subject to} \\
&\sum_{j=1}^{n} x'_{ij}\lambda'_j \leqq \phi x'_{io}, \quad i = 1, 2, \ldots, |M \setminus F|, \\
&\sum_{j=1}^{n} \delta^*_{f'j}\lambda'_j \leqq \phi \delta^*_{f'o}, \quad f' = 1, 2, ..., |F \setminus \{\}|, \\
&\sum_{j=1}^{n} y'_{rj}\lambda'_j \geqq y'_{ro}, \quad r = 1, 2, \ldots, |S \setminus G|, \\
&\sum_{j=1}^{n} \delta^*_{g'j}\lambda'_j \geqq \delta^*_{g'o}, \quad g' = 1, 2, ..., |G \setminus \{\}|, \\
&\sum_{j=1}^{n} \lambda'_j = 1, \\
&\lambda'_j \geqq 0, j = 1, 2, \ldots, n.
\end{aligned}
\tag{11}
$$

Figure 7 graphically summarizes the input and output grouping rationale for an input-oriented model. For example, let there be seven inputs and nine outputs and let us say that as per the experts' opinion, the seven inputs could be grouped into $M = \{\{1, 2, 3\}, \{4, 6\}, \{5\}, \{7\}\}$, then $F = \{\{1, 2, 3\}, \{4, 6\}\}$ and $M \setminus F = \{\{5\}, \{7\}\}$. Similarly, the nine outputs could be grouped into $S = \{\{1\}, \{2, 4\}, \{3\}, \{5, 6, 7\}, \{8, 9\}\}$, then $G = \{\{2, 4\}, \{5, 6, 7\}, \{8, 9\}\}$ and $S \setminus G = \{\{1\}, \{3\}\}$. In this case, also, in order to collapse every subset of F and G, one has to use Model (2) repeatedly as many times as the number of subsets in $F \setminus \{\}$ and $G \setminus \{\}$, respectively.

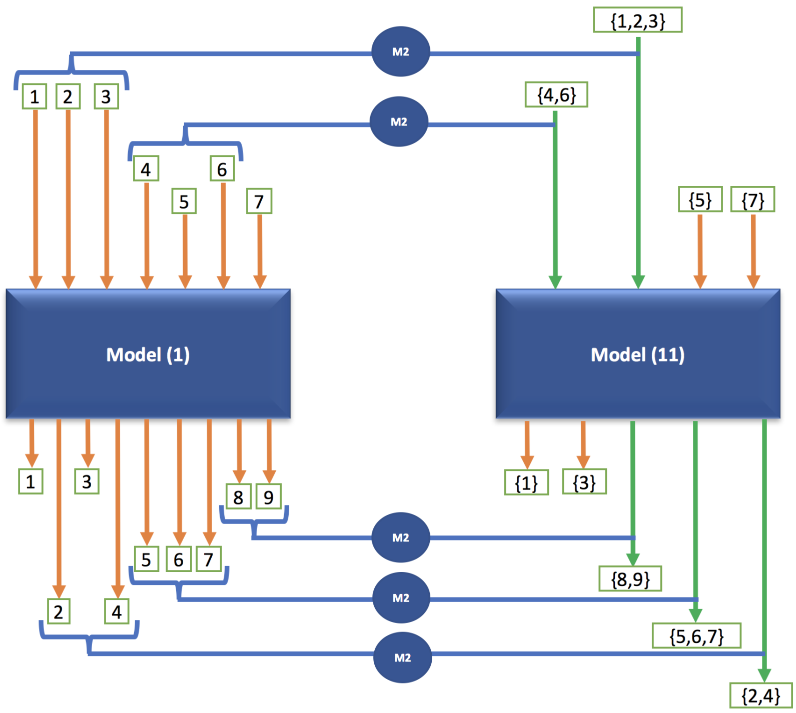

Figure 7: Collapsing inputs and outputs into fewer inputs and outputs.

$$
\begin{aligned}
&\max \varphi \\
&\text{subject to} \\
&\sum_{j=1}^{n} x'_{ij}\lambda'_j \leqq x'_{io}, \quad i = 1, 2, \ldots, |M \setminus F|, \\
&\sum_{j=1}^{n} \delta^*_{f'j}\lambda'_j \leqq \delta^*_{f'o}, \quad f' = 1, 2, ..., |F \setminus \{\}|, \\
&\sum_{j=1}^{n} y'_{rj}\lambda'_j \geqq \varphi y'_{ro}, \quad r = 1, 2, \ldots, |S \setminus G|, \\
&\sum_{j=1}^{n} \delta^*_{g'j}\lambda'_j \geqq \varphi \delta^*_{g'o}, \quad g' = 1, 2, ..., |G \setminus \{\}|, \\
&\sum_{j=1}^{n} \lambda'_j = 1, \\
&\lambda'_j \geqq 0, j = 1, 2, \ldots, n.
\end{aligned} \tag{12}
$$

Both Model (11) and Model (12) have $|M| + |S| + 1$ constraints, out of which $|M|$ in-put constraints, $|S|$ output constrains, $|F \setminus \{\}|$ input grouping constraints, and $|G \setminus \{\}|$output grouping constraints. For the input-oriented envelopment model with both in-put and output grouping (Model (11)), the discriminant score of the j^{th} DMU is ob-tained by Equation (5), wherein $\delta^*_j = max(\delta^*_{g'j}, g' = 1, 2, ..., |G \setminus \{\}|), \forall j$. On the other hand, for the output-oriented envelopment model with both input and output grouping (Model (12)), the discriminant score of the j^{th} DMU is obtained by Equation (9), wherein $\delta^*_j = max(\delta^*_{f'j}, f' = 1, 2, ..., |F \setminus \{\}|), \forall j$.

We shall now examine three real cases to illustrate the applicability of the proposed approach. In the first case, we collapse the outputs; in the second case, we collapse the inputs; and finally, in the third case, we collapse the number of variables in both the inputs and outputs side. In all the three cases, at least one of the thumb rules identified in the in-troduction section is not complied with, which indicates the existence of the dimensionality curse. This further translates into a weakened discriminatory power between efficient and inefficient DMUs, evidenced, as we shall see, by the classification of most of the DMUs as efficient. Just as a reminder, Model (2) represents the base model in our approach, which we use to collapse the inputs and/or outputs.

4. Cases

4.1. Case I: Printed Circuit Boards

Let us consider the case of a teleprinter-manufacturing company that assembles printed circuit boards (PCBs). A PCB is a device that mechanically supports and electrically connects electronic components using conductive tracks, pads, and other features etched from copper sheets laminated onto a non-conductive substrate. Components such as capacitors, resistors, or active devices are generally soldered on the raw PCB. In general, the assembly of PCBs can be a very long process, as it involves a high number of components and solder joints in the products, which can result in several errors.

There are 11 types of defects that can contribute to a defective assembled PCB. These are: wrong component assembled (WCA), reversal component (RC), component missing (CM), wrong cut done/cut not done (WCD/CND), pattern cut (PC), pin bend in ICs (PB), dry soldering (DS), not cleaned (NC), wrong strapping done (WSD), not mounted properly (NMP), and solder short (SS). These 11 defects can be classified into three kinds of errors, that is, machine errors (DS and SS), manual errors (WCA, RC, CM, WCD/CND, PB, and WSD), and other errors (PC, NC, and NMP).

Given the high demands placed on quality, the company is interested in evaluating the efficiency of its assembled PCBs, which could further help the management in working out appropriate interventions to prevent failures. The company processes 38 types of PCBs, which are being assembled in four different assembly units: 35 types of PCBs are processed in one single assembly unit and three types of PCBs are assembled in more than one assembly unit. When a PCB passes through more than one assembly unit, it is considered as a different PCB in each of the respective units. As such, the company manages a total of 43 types of PCBs, as follows: assembly unit 1 processes 21 types of PCBs, assembly unit 2 processes 13 types of PCBs, assembly unit 3 processes 8 types of PCBs, and assembly unit 4 processes 1 type of PCB. For further details regarding the data set, please refer to the paper by Charles, Kumar, and Irene Kavitha (2012).

We have one input and four outputs, out of which one is desirable and three are undesirable. The input is represented by the number of raw PCBs (x_1); the desirable output by the number of assembled PCBs free from all errors (y_d); and the three undesirable outputs by the number of machine errors (y_{u1}), manual errors (y_{u2}), and other errors (y_{u3}).

We are concerned with assembly unit 3, which fails to discriminate between efficient and inefficient DMUs. The data concerning the input and outputs of assembly unit 3 are provided in Table 5. It can be observed from the θ column that there are lots of DMUs that are deemed to be efficient (seven out of the eight DMUs).

Table 5: Discriminatory Power among various types of PCB

DMU	x_1	y_d	y_{u1}	y_{u2}	y_{u3}	θ	δ^*	ϕ	D_S	θ_w	ϕ_w	D_S^w
LCC	49	32	11	10	2	0.9416	0.5543	0.8945	0.4958	1.0000	0.9927	0.5502
TSC	553	478	13	37	28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
PSUI	47	23	18	2	3	1.0000	0.7343	0.7285	0.5349	1.0000	1.0000	0.7343
PSUII	69	42	26	1	1	1.0000	1.0000	0.7896	0.7896	1.0000	1.0000	1.0000
TTC	20	8	2	10	2	1.0000	0.2703	1.0000	0.2703	1.0000	1.0000	0.2703
TUC	54	31	9	21	1	1.0000	0.6291	0.7919	0.4982	1.0000	0.9118	0.5736
TUI	275	249	6	16	2	1.0000	0.4468	1.0000	0.4468	1.0000	1.0000	0.4468
VCP	14	4	3	7	9	1.0000	0.3214	1.0000	0.3214	1.0000	1.0000	0.3214

All the three types of errors have been used to obtain δ^* using Model (2). The θ column in Table 5 has been obtained using Model (1), by means of considering three undesirable outputs as uncontrollable inputs; hence, Model (1) has been run with one input, three un-controllable inputs, and one desirable output. ϕ has been obtained based on Model (4), by incorporating a desirable output constraint. We have also considered the weak disposability assumption of the undesirables; hence, θ_w and ϕ_w have been obtained and, in line with Equation (5), the discriminant score (D^w) is the product of ϕ_w and δ^*. Based on column D_S (=$\phi\delta^*$), it can be observed that card TSC is the only efficient one, whereas under the weak disposability assumption, only two cards, that is, TSC and PSUII, are efficient (see the last column of Table 5). In other words, by means of using our proposed approach we have been able to increase the discriminatory power among the eight individual cards.

4.2. Case II: Greek Banks

For this second application, let us consider the case of ten commercial non-core Greek banks, for which data has been collected from the database of the HBA (2011). Given that the banking industry plays a vital role in the economy, it is of interest to evaluate the efficiency of the banks.

In this case, we have three inputs, namely inputs-operating expenses or OPEX (x_1), loan loss provisions or LLP (x_2), and haircut on Greek bonds held by the banks or PSI (x_3). We also have one single output, namely total loans or LOANS (y_1). OPEX is calculated as the sum of all expenses reported by the banks; LLP is weakly disposable and is measured as the portion of a bank's cash or cash-equivalent holdings set aside as an allowance for uncollected loans and loan payments; and PSI is an uncontrollable input that represents the loss incurred by each bank from the exchange of Greek Government Bonds for a series of new bonds, at a significant price discount. LOANS, on the other hand, is calculated as the sum of all loan accounts intermediated by the banks. For more information regarding the choice and definition of inputs/output, the reader is referred to the paper by Tsolas and Charles (2015).

Let us thus evaluate the efficiency of the ten banks. The data concerning the inputs and output of banks are provided in Table 6. In this case, also, the ψ column indicates that most of the DMUs are deemed to be efficient (nine out of the ten DMUs).

Table 6: Discriminatory Power among commercial non-core Greek banks

DMU	$OPEX$	LLP	PSI	$LOANS$		δ^{*}	φ^{-1}	D_S
Emporiki	520.2	1100	592.0	19135	1.0000	1.0000	1.0000	1.0000
ATE	340.5	1241.8	2163	18450	1.0000	1.0000	0.6546	0.6546
Millenium	135.0	89.2	173.1	4744.2	1.0000	0.0800	1.0000	0.0800
Geniki	135.6	462.4	287.6	3172.4	1.0000	0.4170	0.4911	0.2048
Attica	112.0	253.0	142.1	3579.9	1.0000	0.2295	0.5669	0.1301
Probank	85.7	59.0	334.4	2721	1.0000	0.1546	0.5156	0.0797
Nea Proton	11.7	91.6	146.5	923	1.0000	0.0749	0.0925	0.0069
FFB	29.2	93.3	49.1	1400.3	0.6560	1.0000	0.0561	0.0561
Panellhnia	21.0	13.8	19.2	588.1	1.0000	0.0115	1.0000	0.0115
ABB	6.9	0.6	6.7	269.2	1.0000	0.0031	1.0000	0.0031

ψ has been obtained based on Model (7); furthermore, the three inputs OPEX, LLP, and PSI have been used to obtain δ^{*} using Model (2). It is to be noted that one has to meaningfully alter Model (2) and Model (7) in accordance with the information associated with the inputs. The φ (so as φ^{-1}) is obtained based on Model (8). The last column of Table 6 shows that Emporiki bank is efficient, while the least efficiency score is associated with the ABB bank. Once again, our proposed approach was successfully used to discriminate among the efficient and inefficient banks.

4.3. Case 3: Quality of Life in Fortune's Best Cities

Finally, let us consider a more complex situation, to demonstrate the grouping of inputs and outputs at the same time; for this case, we refer to the data set of *Fortune* Magazine's 20 best cities in 1996 (comprising 15 US domestic cities and five international cities), as reported in the paper by Zhu (2001). To select its best cities, *Fortune* Magazine uses factors that measure aspects of the cost of living, demographics, business, and leisure.

Based on these factors, Zhu (2001) developed 6 inputs and 6 outputs. The 6 inputs are represented by the high-end housing price (x_1), lower-end housing monthly rental (x_2), the cost of a loaf of French bread (x_3), the cost of martini (x_4), Class A office rental (x_5), and the number of violent crimes (x_6). The 6 outputs are the median household income (y_1), number of population with bachelor's degree (y_2), number of doctors (y_3), number of museums (y_4), number of libraries (y_5), and number of 18-hole golf courses (y_6).

The following Table 7 provides the data for the 6 inputs and 6 outputs. The last but one column E represents the VRS scores $\theta_o^* - \epsilon(\sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+)$, where θ_0^* is the efficiency score of the DMU of interest, s_i^- and s_r^+ are the input and output slacks, respectively, and ϵ is the non-Archimedean. The last column E_s represents the slack-adjusted VRS scores, computed based on $\theta_o^* - \frac{1}{m}(\sum_{i=1}^m \frac{s_i^-}{x_{io}})$.

Table 7: Inputs and outputs of Fortune's Best Cities with efficiency scores

DMU	x_1	x_2	x_3	x_4	x_5	x_6	y_1	y_2	y_3	y_4	y_5	y_6	E	E_s
Seattle	586000	581	1.45	4.50	21	542.3	46928	0.297	4.49	7	117	22	1.00	1.00
Denver	475000	558	0.97	4.00	14	595.6	42879	0.291	2.79	5	60	71	1.00	1.00
Philadelphia	201000	600	1.50	4.75	21	693.6	43576	0.227	3.64	25	216	166	1.00	1.00
Minneapolis	299000	609	1.49	4.00	24	496.5	45673	0.270	2.67	6	131	125	1.00	1.00
Ral-Durham	318000	613	0.99	4.50	18	634.7	40990	0.319	4.94	7	33	47	1.00	1.00
St. Louis	265000	558	0.89	3.00	18	263.0	39079	0.206	3.40	10	104	62	1.00	1.00
Cincinnati	467000	580	1.25	3.75	20	551.5	38455	0.199	2.80	4	71	94	1.00	1.00
Washington	583000	625	1.29	3.75	33	714.5	54291	0.373	3.35	30	148	105	1.00	1.00
Pittsburgh	347000	535	0.99	3.75	17	382.1	34534	0.188	3.66	8	124	112	1.00	1.00
Dallas–FW	296000	650	1.50	5.00	18	825.4	41984	0.271	1.96	3	98	77	1.00	1.00
Atlanta	600000	740	1.19	6.75	20	846.6	43249	0.263	2.23	9	118	102	0.98	0.80
Baltimore	575000	775	0.99	3.99	18	1296.3	43291	0.233	4.02	8	102	45	1.00	1.00
Boston	351000	888	1.09	4.25	34	686.6	46444	0.325	5.69	25	240	55	1.00	1.00
Milwaukee	283000	727	1.53	3.50	26	518.9	41841	0.214	3.11	6	52	50	1.00	1.00
Nashville	431000	695	1.19	4.00	26	1132.5	40221	0.215	3.25	4	37	37	0.81	0.74

We proceed to evaluate the efficiency of the 15 US domestic cities from the data set under analysis. In this case, also, we find ourselves in a situation in which an immediate efficiency calculation deems many cities to be efficient, when this may not necessarily be the case (as a matter of fact, 18 cities out of 20 are cased as efficient, see the last two columns in Table 7; also, see Zhu (2001)).

In line with our proposed approach, we could once again proceed to collapse the inputs or outputs side, to try to reduce the number of variables. What we can observe in this case, however, is that inputs and outputs can be meaningfully collapsed simultaneously. As per the *Fortune* Magazine's data set presented in Zhu (2001), the 6 inputs can be logically grouped into 3 sets, as follows: $\{x_1, x_2, x_3, x_4\}$ refer to Cost of Living (CoL), $\{x_5\}$ refers to Business Leisure (BL_I), and $\{x_6\}$ refers to Quality of Life (QoL_I). In a similar fashion, the 6 outputs can also be grouped into 3 sets, as follows: $\{y_1, y_2\}$ refer to Demographics ($Demo$), $\{y_3\}$ refers to Quality of Life (QoL_O), and $\{y_4, y_5, y_6\}$ refer to Business Leisure (BL_O). Hence, let $S = \{\{x_1, x_2, x_3, x_4\}, \{x_5\}, \{x_6\}, \{y_1, y_2\}, \{y_3\}, \{y_4, y_5, y_6\}\} = \{CoL, BL_I, QoL_I, Demo, QoL_O, BL_O\}$, $G = \{\{x_1, x_2, x_3, x_4\}, \{y_1, y_2\}, \{y_4, y_5, y_6\}\}$, wherein CoL, $Demo$, and BL_O have been obtained using Model (2). In consequence, we now have a total of $3 + 3 = 6$ inputs and outputs (see columns 2-7 in Table 8).

Table 8: Discriminatory Power among Fortune's Best Cities

DMU	CoL	BL_I	QoL_I	$Demo$	QoL_O	BL_O	E	δ^*	φ^{-1}	D_S
Seattle	1.000	21	542.3	0.864	4.49	0.488	1.00	0.864	1.000	0.864
Denver	0.794	14	595.6	0.790	2.79	0.428	1.00	0.790	1.000	0.790
Philadelphia	0.997	21	693.6	0.803	3.64	1.000	1.00	1.000	1.000	1.000
Minneapolis	0.983	24	496.5	0.841	2.67	0.753	1.00	0.841	1.000	0.841
Ral.-Durham	0.800	18	634.7	0.855	4.94	0.283	1.00	0.855	1.000	0.855
St. Louis	0.700	18	263	0.720	3.40	0.466	1.00	0.720	1.000	0.720
Cincinnati	0.891	20	551.5	0.708	2.80	0.566	1.00	0.708	0.843	0.597
Washington	0.983	33	714.5	1.000	3.35	1.000	1.00	1.000	1.000	1.000
Pittsburgh	0.737	17	382.1	0.636	3.66	0.675	1.00	0.675	1.000	0.675
Dallas–FW	1.000	18	825.4	0.773	1.96	0.464	1.00	0.773	0.802	0.620
Atlanta	1.000	20	846.6	0.797	2.23	0.614	0.98	0.797	0.851	0.678
Baltimore	1.000	18	1296.3	0.797	4.02	0.444	1.00	0.797	0.941	0.750
Boston	1.000	34	686.6	0.871	5.69	1.000	1.00	1.000	1.000	1.000
Milwaukee	1.000	26	518.9	0.771	3.11	0.301	1.00	0.771	0.742	0.571
Nashville	0.923	26	1132.5	0.741	3.25	0.223	0.81	0.741	0.775	0.574

By means of carrying forward with our approach, the φ (so as φ^{-1}) is obtained based on Model (12). From the last column of Table 8, we can observe that actually only 3 cities are efficient: Philadelphia, Washington, and Boston; with Nashville, Milwaukee, and Cincinnati being the least efficient ones. It is thus shown that our proposed approach can calculate a discriminant score that helps to discriminate among the efficient and inefficient cities.

5. Conclusion

Performance evaluation is an important activity in the process of identifying shortcomings in managerial efficiency and devising goals and strategies for improvement (Morita & Avkiran, 2009). One of the most popular techniques to evaluate the performance is DEA; and although this technique has proven useful in various fields throughout the years, sup-porting decision-making worldwide (Charles, Tsolas, & Gherman, 2018), the selection of inputs and outputs has been a constant concern. As it is well known, DEA is sensitive to such variable selection in the sense that, the more variables added, the greater is the chance for some inefficient units to dominate the added dimension and be classified as efficient (Smith, 1997). Otherwise stated, when the number of DMUs is below the empirical thresh-old levels proposed in the literature that relate the number of variables with the number of observations, the discriminatory power between efficient and inefficient DMUs may drastically weaken; in consequence, performance evaluation may be affected. In the literature, the lack of discrimination is often referred to as the "curse of dimensionality".

In this paper, we have provided a simple approach using the well-known pure DEA model to increase the discriminatory power between efficient and inefficient DMUs. We have shown how inputs only or outputs only can be collapsed into a single input (or multiple inputs) and single output (or multiple outputs), respectively; and how the collapse can also simultaneously take place in both the inputs and outputs side. In all the cases, it was possible to avoid having to meet the empirical rules of thumb regarding the number of DMUs relative to the number of inputs and outputs.

In terms of limitations of the present research, as in the standard DEA, we assume that the input and output data are deterministic and non-statistical. If one is interested in adopting the current approach for stochastic DEA, we could assume that the data are generated from a population through a data-generating process. However, such an assumption may require significant new development and we view this as a potential future research topic.

Lastly, regarding the input and output grouping rationale, in this paper we have considered that the subsets (groups) are defined by the experts and that opinions are derived in a qualitative way. Nevertheless, such opinions could also be derived in a quantitative way and we position this as an avenue for future research. Furthermore, in this paper, we have discussed radial DEA only; future research could also consider various non-radial approaches.

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References

1. Adler, N., & Golany, B. (2001). Evaluation of deregulated airline networks using data envelopment analysis combined with principal component analysis with an application to Western Europe. European Journal of Operational Research, 132(2), 260-273.

2. Adler, N., & Golany, B. (2002). Including principal component weights to improve discrimination in data envelopment analysis. Journal of the Operations Research Society, 53(9), 985-991.

3. Adler N., & Golany B. (2007). PCA-DEA: Reducing the curse of dimensionality. In J. Zhu, & W. D. Cook (Eds.), Modeling Data Irregularities and Structural Complexities in Data Envelopment Analysis (pp. 139-153). Boston, MA: Springer.

4. Adler, N. & Yazhemsky, E. (2010). Improving discrimination in data envelopment analysis: PCA-DEA or variable reduction. European Journal of Operational Research, 202(1), 273-284.

6. Allen, R., Athanassopoulos, A., Dyson, R. G., & Thanassoulis, E. (1997). Weights restrictions and value judgments in data envelopment analysis: evolution, development and future directions. Annals of Operational Research, 73, 13-34.

7. Andersen, P., & Petersen, N. C. (1993). A procedure for ranking efficient units in data envelopment analysis. Management Science, 39, 1261-1264.

8. Banker, R.D., Charnes, A., & Cooper, W.W. (1984). Some models for estimating technical and scale inefficiencies in Data Envelopment Analysis. Management Science, 30, 1078-1092.

9. Banker, R. D., Charnes, A, Cooper, W. W., Swarts, J., & Thomas, D. A. (1989). An introduction to data envelopment analysis with some of its models and their uses. In J. L. Chan, & J. M. Patton (Eds.), Research in Governmental and Nonprofit Accounting (pp. 125-163). Connecticut, CT: Jai Press.

10. Charles, V., Kumar, M., & Irene Kavitha, S. (2012). Measuring the efficiency of assembled printed circuit boards with undesirable outputs using data envelopment analysis. International Journal of Production Economics, 136, 194-206.

11. Charles, V., Tsolas, I. E., & Gherman, T. (2018). Satisficing data envelopment analysis: a Bayesian approach for peer mining in the banking sector. Annals of Operations Research, 269(1-2), 81-102.

12. Cherchye, L., Moesen, W., & Puyenbroeck, T. (2004). Legitimately diverse, yet comparable: on synthesizing social inclusion performance in the EU. JCMS: Journal of Common Market Studies 42(5): 919-955.

13. Cherchye, L., Lovell, C. K., Moesen, W., & Van Puyenbroeck, T. (2007). One market, one number? A composite indicator assessment of EU internal market dynamics. European Economic Review 51(3): 749-779.

14. Cinca, C., & Molinero, C. M. (2004). Selecting DEA specifications and ranking units via PCA. Journal of the Operational Research Society, 55(5), 521-528.

15. Cook, W. D., Tone, K., & Zhu, J. (2014). Data envelopment analysis: prior to choosing a model. Omega: The International Journal of Management Science, 44, 1-4.

16. Cooper, W. W., Seiford, L. M., & Tone, K. (2007). Data envelopment analysis: a comprehensive text with models, applications, references and DEA-solver software (2nd ed.). New York, NY: Springer.

17. Daraio, C., & Simar, L. (2007). Advanced robust and nonparametric methods in efficiency analysis: Methodology and applications. New York, NY: Springer.

18. Doyle, J. R., & Green, R. (1994). Efficiency and cross-efficiency in data envelop-ment analysis: derivatives, meanings and uses. Journal of the Operational Research Society, 45(5), 567-578.

19. Dyson, R. G, Allen, R., Camanho, A., Podinovski, V., Sarrico, C., & Shale, E. (2001). Pitfalls and protocols in DEA. European Journal of Operational Research, 132(2), 245-259.

20. Friedman, L., & Sinuany-Stern, Z. (1998). Combining ranking scales and selecting variables in the DEA context: The case of industrial branches. Computers and Operations Research, 25(9), 781-791.

21. Geerts, G. L. (2011). A design science research methodology and its application to accounting information systems research. International Journal of Accounting Information Systems, 12, 142-151.

22. Ghasemi, M. R., Ignatius, J., & Rezaee, B. (2019). Improving discriminating power in data envelopment models based on deviation variables framework. European Journal of Operational Research, 278, 442-447.

23. Golany, B., & Roll, Y. (1989). An application procedure for DEA. Omega: The International Journal of Management Science,17(3), 237-250.

24. HBA (2011). The Greek banking system in 2010. Athens, Greece: Hellenic Bank Association. (in Greek) [online], [retrieved 14 June 2011]. Retrieved from: http:// www.hba.gr.

25. Hevner, A. R., March, S. T., Park, J., & Ram, S. (2004). Design science in information systems research. MIS Quarterly, 28(1), 75-105.

26. Homburg, C. (2001). Using data envelopment analysis to benchmark activities. International Journal of Production Economics, 73(1), 51–58.

27. Hughes, A., & Yaisawarng, S. (2004). Sensitivity and dimensionality tests of DEA efficiency scores. European Journal of Operational Research, 154, 410-422.

28. Jenkins, L., & Anderson, M. (2003). A multivariate statistical approach to reducing the number of variables in data envelopment analysis. European Journal of Operational Research, 147, 51-61.

29. Liang, L., Li, Y., & Li, S. (2009). Increasing the discriminatory power of DEA in the presence of the undesirable outputs and large dimensionality of data sets with PCA. Expert Systems with Applications, 36, 5895-5899.

30. Lovell, C. K., & Pastor, J. T. (1999). Radial DEA models without inputs or without outputs. European Journal of Operational Research, 118(1), 46-51.

31. Meng, W., Zhang, D. Q., Qi, L., & Liu, W. B. (2008). Two-level DEA approaches in research evaluation. Omega, 36(6), 950-957.

32. Morita, H., & Avkiran, N. K. (2009). Selecting inputs and outputs in data envelopment analysis by designing statistical experiments. Journal of the Operations Research Society of Japan, 52(2), 163-173.

33. Nunamaker, T. R. (1985). Using data envelopment analysis to measure the efficiency of non-profit organizations: A critical evaluation. Managerial and Decision Economics, 6(1), 50-58.

34. Pastor, J. T., Ruiz, J. L., & Sirvent, I. (2002). A statistical test for nested radial DEA models. Operations Research, 50, 728-735.

35. Peffers, K., Tuunanen, T., Rothenberger, M. A., & Chatterjee, S. (2008). A design science research methodology for information systems research. Journal of Management Information Systems, 24(3), 45-77.

36. Raab, R., & Lichty, R. (2002). Identifying sub-areas that comprise a greater metropolitan area: The criterion of country relative efficiency. Journal of Regional Science, 42(3), 579-594.

37. Ragsdale, C. T. (2006). Spreadsheet Modeling and Decision Analysis (3rd ed., p. 132). Cincinnati, OH: Thomson Nelson.

38. Rezaeiani, M. J., & Foroughi, A. A. (2018). Ranking efficient decision making units in data envelopment analysis based on reference frontier share. European Journal of Operational Research, 264, 665-674.

39. Sarkis, J. (2000). A comparative analysis of DEA as a discrete alternative multiple criteria decision tool. European Journal of Operational Research, 123, 543-557.

40. Seiford, L. M., & Zhu, J. (1998). An acceptance system decision rule with data envelopment analysis. Computers and Operations Research, 25(4), 329-332.

41. Seiford, L. M., & Zhu, J. (1999). Infeasibility of super-efficiency data envelopment analysis. INFOR: Information Systems and Operational Research, 37(2), 174-187.

42. Shen, W. F., Zhang, D. Q., Liu, W. B., & Yang, G. L. (2016). Increasing discrimination of DEA evaluation by utilizing distances to anti-efficient frontiers. Computers and Operations Research, 75, 163-173.

43. Smith, P. (1997). Model misspecification in data envelopment analysis. Annals of Operations Research,73(1), 233-252.

44. Tsolas, I. E., & Charles, V. (2015). Incorporating Risk into Bank Efficiency: A Satisficing DEA Approach to Assess the Greek Banking Crisis. Expert Systems with Applications, 42, 3491-3500.

45. Wagner, J. M., & Shimshak, D. G. (2007). Stepwise selection of variables in data envelopment analysis: Procedures and managerial perspectives. European Journal of Operational Research, 180, 57-67.

46. Wieringa, R. J. (2014). Design Science Methodology for Information Systems and Software Engineering. New York, NY: Springer-Verlag.

47. Wong, Y. H. B, & Beasley, J. E. (1990). Restricting weight flexibility in data envelopment analysis. Journal of the Operational Research Society, 41, 829-835.

48. Wu, J., Liang, L., & Chen, Y. (2009). DEA game cross-efficiency approach to Olympic rankings. Omega, 37, 909-918.

49. Xie, Q., Dai, Q., Li, Y., & Jiang, A. (2014). Increasing the discriminatory power of DEA using Shannon's entropy. Entropy, 16, 1571-1582.

50. Zhang, D. Q., Li, X. X., Meng, W., & Liu, W. B. (2009). Measure the performance of nations at Olympic Games using DEA models with different preferences. Journal of the Operational Research Society, 60, 983-990.

51. Zhu, J. (2001). Multidimensional quality-of-life measure with an application to Fortune's best cities. Socio-Economic Planning Sciences, 35, 263-284.